Problem IV.1 ... the flight over the moon

3 points; průměr 2,53; řešilo 118 studentů

One day, the FYKOS-bird was watching the sky during a full moon. An airplane just passed over the moon in 0.35 s*, and the perpendicular distance of its flight path from the center of the moon was* 1/3 *of the full moon's radius. This plane flies typically with a speed of* $800\,\mathrm{km\cdot h^{-1}}$*. The FYKOS-bird wondered what altitude the plane was at so he could fly with it next time. Like him, determine this altitude.* *Jarda was sunbathing in the garden.*

Let us denote the diameter of the Moon as $d \doteq 3\,475\,\mathrm{km}$ and its distance from the Earth as $R \doteq \doteq 384\,000\,\mathrm{km}$. However, we must convert the diameter to the distance on the Moon d' that the plane will fly over (at a distance of 1/3 from the center of the Moon). We use the Pythagorean theorem to get

$$\left(\frac{d}{2}\right)^2 = \left(\frac{1}{3}\cdot\frac{d}{2}\right)^2 + \left(\frac{d'}{2}\right)^2 \quad \Rightarrow \quad d' = \frac{\sqrt{8}}{3}d\,.$$

Using the velocity v and the time t we calculate the path of the plane s

$$s = vt \doteq 78\,\mathrm{m}\,,$$

We assume that the triangle with a peak on the Earth's surface and opposite side formed by the plane's path traveled in time t has the sought height h and is similar to the triangle Earth's surface and length d' on the Moon, which has as its height the Moon-Earth distance. Then

$$\frac{R}{d'} = \frac{h}{s} \quad \Rightarrow \quad h = \frac{Rs}{d'} = \frac{3Rvt}{\sqrt{8}d} = \frac{3\cdot 384\,400\,\mathrm{km}\cdot 222\,\mathrm{m\cdot s^{-1}}\cdot 0.35\,\mathrm{s}}{\sqrt{8}\cdot 3\,475\,\mathrm{km}} \doteq 9\,100\,\mathrm{m}$$

therefore the plane is at the height where planes usually fly.

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