Problem I.E ... wipe the paper

12 points; průměr 7,48; řešilo 93 studentů

Measure the coefficient of static friction between two sheets of office paper.

Karel browsed through the recipes.

In this experiment, we will attempt to measure the coefficient of friction between two sheets of office paper in domestic conditions. Friction is generally measured using roughness testers or tribometers, which can vary in sophistication. We will employ the simplest method, measuring on an inclined plane.

Theory

The decomposition of forces acting on an object on an inclined plane is one of the fundamental problems in high school physics. Here, we will briefly recall the procedure and results. Three forces act on the object: gravitational force, the ground reaction force, and the frictional force. The gravitational force acts downward with a magnitude of $F_g = mg$ where m is the mass of the object, and $g = 9.81\,\mathrm{m{\cdot}s^{-1}}$ is the gravitational acceleration. The reaction force prevents the object from sinking through the material beneath it. It also causes frictional force between the ground and the object. The frictional force acts against the (possible) motion of the object.

Let's decompose all forces into components perpendicular and parallel to the surface. Consider that the surface makes an angle α with the horizontal line (which is perpendicular to the gravitational acceleration). In the direction perpendicular to the plane, the gravitational force has a component (let's call it the normal component) with a magnitude $F_{g\perp} = N = F_g \cos\alpha = mg\cos\alpha$. It must balance the reaction force because the object does not fall through the surface. Thus, we have solved the forces in the normal direction. In the direction parallel to the inclined plane, the tangential component of the gravitational force acts on the object $T = F_g \sin\alpha = mg\sin\alpha$. This force could cause the object to move downward. However, the opposite direction is acted upon by the frictional force. The magnitude of the frictional force is always equal to or less than the product of the normal force, and the coefficient of friction $F_\mathrm{t} = fN = fmg\cos\alpha$, where f represents the investigated coefficient. As long as the force that sets the object in motion is small enough, the magnitude of the frictional force adjusts to it, and the object does not start accelerating. Otherwise, the magnitude of the frictional force is constant and precisely equal to F_t. The total force acting on the object downward along the inclined plane is $F = ma = mg\left(\sin\alpha - f\cos\alpha\right)$.

The coefficient of friction generally differs between bodies at rest and bodies in motion. In our experiment, we will gradually increase the angle of inclination α until the tangential component of the gravitational force exceeds the frictional force and the object starts moving. Therefore, we will measure the coefficient of static friction by the given task. Once the object starts moving, we will stop increasing the inclination angle, and we will record the angle α as the critical angle α_m. At that moment, we have reached the condition under which approximately the following holds:

$$F = ma = mg\left(\sin\alpha_\mathrm{m} - f\cos\alpha_\mathrm{m}\right) = 0\,,$$

from where we can, even without knowledge about gravitational acceleration and mass, determine the coefficient of friction as

$$mg\left(\sin\alpha_\mathrm{m} - f\cos\alpha_\mathrm{m}\right) = 0 \quad\Rightarrow\quad f = \frac{\sin\alpha_\mathrm{m}}{\cos\alpha_\mathrm{m}} = \tan\alpha_\mathrm{m}\,.$$

Measuring critical angle α_m gives us the wanted coefficient of friction.

Arrangement and execution of the experiment

We already briefly described the measurement procedure in the previous section. We will use a wooden board with a size similar to the dimensions of an A3 paper as the inclined plane. We will modify the board's surface to measure the coefficient of friction between two sheets of office paper. We will attach two sheets of office paper using adhesive tape, covering almost the entire board. Then, we will place a sheet of paper on the board parallel to its longer dimensions and start raising it. We lift the board from a low angle where the laid paper still holds without problems. The lifting is done by inserting a rigid object (a square-shaped flowerpot) under the board and lifting it. We secure the second bottom side of the board with a heavy object (clamp). Once the paper starts sliding, we stop inserting the rigid object and measure the critical inclination angle. After that, we retract the object to reduce the angle α sufficiently, place the paper on the inclined plane a bit higher, and repeat the measurement.

The angle of inclination of the board is measured using a mobile phone, which is attached directly to the board using adhesive tape. The numerical value of the angle is obtained using the *phyphox* application, in the *Tilt* tool, in the *Plane* section. The application should be able to determine the inclination of the board relative to the horizontal plane, regardless of the phone's orientation on the inclined plane. In our experiment, the phone is fixed in the same direction, allowing us to compare the measured values without any worry or concern. Before the actual measurement, we measured the length of the board and the distance from its upper side to the ground, from which we obtained the angle of inclination and compared it with the value from the mobile phone. The data did not differ significantly, verifying the reliability of the values measured by the *phyphox* application within the chosen interval of precision. The application displays the result to a tenth of a degree, which is significantly more accurate than all other tools in domestic conditions. We will discuss the precision of the measured values in the *Results* and *Discussion* sections. For a better statistical data set (and thanks to the low demands of the experiment), we measured the critical angle of inclination α_{m} for three sheets of office paper. Two came from the printer's stack, and one was taken from a classic 500-sheet package of office paper just before the experiment. We will see if the coefficient of friction depends on the paper's history. We did not replace papers attached to the board during the measurement. For curiosity, we wrapped a heavy object (head of a hammer) in the same office paper (from the printer) and repeated the experiment. We did that because light sheets of paper may be affected by additional factors, leading to a different coefficient of friction. In the measurement with the hammer, we measured the coefficient of friction of paper on paper, but we cannot claim to have measured the coefficient of friction between two sheets of office paper. Below, we attach a photograph 1 of the experimental setup along with the measured objects.

Results

In this chapter, we will present the measured values of α_{m} for all three sheets of office paper and the head of a hammer wrapped in paper. We conducted fourteen measurements for each of these four objects, with seven measurements performed on one side of the paper and the remainder on the other (including the head). That was necessary to avoid, for example, staining one side and influencing the measured data. From these values, we will determine the coefficient of friction and establish its error. The measured values are in the table 1 below. The application displayed the measured angle to two decimal places. Still, we decided to round them to half a degree each time because the values on the last displayed digit changed rapidly, and approximately a tenth

Fig. 1: Photo of experiment setup. A sheet of office paper on an inclined plane, the head of a hammer wrapped in paper, and a mobile phone. Rigid object in the back – flower pot.

of a degree was not constant during the measurement. We rounded the results because we could not be sure of higher accuracy.

We will convert all critical angles to coefficients of friction using the derived relationship from the theoretical part $f = \tan \alpha_\mathrm{m}$. From these values, we will then calculate, for each object, the arithmetic mean and the standard deviation Δf using the traditional well-known formula:

$$\Delta f = \sqrt{\frac{\Sigma_1^n \left(f - \bar{f}\right)^2}{n\,(n-1)}}\,,$$

where $n = 14$ is the number of measurements for one object, and $\bar{f}$ is the average value of the coefficient of friction. In the numerator under the square root, we sum over all 14 measurements. The standard deviation tells us how much individual measurements deviate from their average value. The smaller the standard deviation, the more precise the measurements are. The results of this procedure are presented in table 2. We determined the standard deviation of the critical angle in the same way, as shown in table 1.

When calculating errors, we should also consider the error of the measuring instrument itself. If we were measuring, for example, with a ruler, we would set it as half of the smallest division. The manufacturer should specify the precision for analog and digital instruments. However, in this case, we could not find any information about the precision of the measurement itself. Since the displayed value on the screen is written to four significant figures, we can assume the measurement error will be relatively small.

On the other hand, even in a stable position, the value changes by tenths of a degree. Therefore, we decided to round the angle measurement to half a degree, and we will not consider the instrument error further. Given the domestic conditions and the precision of all other tools, we cannot verify or disprove it.

Tab. 1: Measured values of critical angle α_m for each sheet of paper (denoted by a number in the index) and heavy object (index "t"). Calculated mean value and standard deviation.

measurement	$\frac{\theta_1}{\circ}$	$\frac{\theta_2}{\circ}$	$\frac{\theta_3}{\circ}$	$\frac{\theta_\mathrm{t}}{\circ}$
1	27.5	24.0	21.0	29.0
2	25.0	26.5	25.0	30.0
3	24.0	27.0	22.0	32.5
4	28.0	24.5	24.0	32.5
5	29.5	25.5	22.5	31.5
6	28.5	26.0	21.5	28.5
7	27.0	28.5	22.5	31.5
8	28.0	27.5	28.0	31.0
9	25.5	26.5	25.0	30.0
10	27.5	26.0	24.5	28.0
11	25.5	25.0	26.5	29.5
12	26.0	25.5	21.0	28.5
13	29.5	29.0	25.5	30.0
14	26.0	28.5	26.5	29.0
mean	27.0	26.5	24.0	30.0
std. dev.	0.5	0.5	0.5	0.5

Tab. 2: Coefficient of friction in each measurement for each sheet of paper (denoted by the number in the index) and heavy object (index "t"). Calculated mean value and standard deviation.

measurement	f_1	f_2	f_3	f_t
1	0.520	0.445	0.384	0.554
2	0.466	0.498	0.466	0.577
3	0.445	0.509	0.404	0.637
4	0.531	0.455	0.445	0.637
5	0.565	0.477	0.414	0.612
6	0.543	0.487	0.394	0.543
7	0.509	0.543	0.414	0.612
8	0.531	0.520	0.531	0.600
9	0.477	0.498	0.466	0.577
10	0.520	0.487	0.455	0.531
11	0.477	0.466	0.498	0.565
12	0.487	0.477	0.384	0.543
13	0.565	0.554	0.477	0.577
14	0.487	0.543	0.498	0.554
mean $\bar{f}$	0.51	0.50	0.45	0.58
std. dev. Δf	0.01	0.01	0.01	0.01

We measured the coefficients of friction between sheets of paper as 0.51 ± 0.01 for the first paper on the inclined plane, 0.50 ± 0.01 for the second, and 0.45 ± 0.01 for the third. The average coefficient of friction is then 0.49 ± 0.02, where a larger error represents the variability of individual sheets of paper. The coefficient of friction of paper on paper (measurement with the hammer) turned out to be 0.58 ± 0.01. In the next chapter, we will discuss the measured results.

Discussion of results

Although we tried to avoid all sources of errors, we cannot minimize them in domestic conditions with such a simple setup. The inaccuracy of the measured values could have arisen simply because the paper slid over two sheets, which were placed as close together as possible, but still, a small gap occurred between them. Generally, from experience, we could assume that this inhomogeneity would tend to increase the coefficient of friction. However, the edges of the glued paper may not be perfectly aligned with the board, which could potentially cause the sliding paper to lift, ultimately reducing friction. Using A3 size paper would improve the measurement.

Unfortunately, when moving a heavy object to lift the inclined plane, we did not work on a perfectly smooth floor, so the movement was not smooth but slightly jerky. By tilting the board at a non-constant speed, we created inertial forces in the system connected to it, which could significantly influence the moment of release of the paper from a stable position. Different methods of lifting the board could eliminate this systematic error. However, in domestic conditions, we did not find a way to stop precisely when the paper started to move and measure the angle. We also observed that when the paper started sliding down the inclined plane, it sometimes stopped on it. An increased friction had to act on it, so the coefficient differed slightly with position.

During the measurements, we noticed that preparing the paper under precisely the same initial conditions is difficult. Even a slight push on a laid sheet increased the critical angle. Sometimes, when we laid the paper at a minuscule angle on the board and lifted it, it started to slide. A sheet of paper is so light that it is affected by forces other than friction, as we know from the motion of other objects. Furthermore, the paper is highly flexible, so it may not always lay flat on the board; some parts may be more or less bent. Although we tried to perform the experiment the same way each time, we could not guarantee that the initial conditions were equivalent every time.

The friction values of the first two sheets of paper were similar within the standard deviation, while the friction was about ten percent lower for the third sheet. Here, we must again mention the origin of the sheets, as the first two came from the printer's stack, while the third came directly from the 500-sheet packaging. It was, therefore, less exposed to external influences, probably cleaner, and less crumpled. We see that friction significantly changes depending on the history of the paper sheets. Therefore, after the seventh measurement, we also rotated the papers because we wanted to compensate for the influence of which side we put them on.

Since we suspected that the coefficient of friction between sheets of office paper might be different when increasing the normal force, we decided to try the experiment again to determine the friction directly between the paper as a material. This time, we loaded the paper from the printer with a heavy weight (hammer). The coefficient of friction is about 15 % higher than with sheets of paper from the printer's stack. The paper was more pressed, and other forces did not affect it much here. So, we can say that the coefficient of friction is not independent of the normal force, as we assumed in the theoretical part, but it increases with its growing value. It might be interesting to compare the coefficient of friction for different loads and papers with different histories, which we did not test in this task.

We calculated standard deviations, and for each sheet of paper, the relative errors of the coefficient of friction were around 2 %, which is relatively low. From this information, we could state that we measured the friction for each object quite precisely, but there are noticeable differences in friction between objects. However, we do not know the instrument's error, and the size of the standard deviations is suspiciously small given the domestic equipment and procedure used. It is possible that rounding the angle value to half a degree is still fine, and the instrument's error is even higher, increasing the overall measurement error. However, we did not test this possibility within the experiment.

Given the measured results, it could be interesting to experiment with even more sheets of paper exposed to different and more extreme conditions, and we would probably get a more significant interval of measured coefficients of friction. We could also measure the dependence of the coefficient of friction on the normal force applied to the paper.

Conclusion

We measured the critical angle at which the paper spontaneously started sliding on the office paper's surface; from that, we calculated the coefficient of friction. The measurements were performed for three sheets of paper with the results of 0.51 ± 0.01 for the first, 0.50 ± 0.01 for the second, and 0.45 ± 0.01 for the third paper on the inclined plane. It is evident from the results that the coefficient of friction is affected by the paper's history and its current condition. Even very subtle influences can significantly alter friction. The average coefficient of friction between the sheets of paper is 0.49 ± 0.02.

Therefore, we also measured the coefficient of friction between sheets of paper, where we loaded the sheet with the head of the hammer. The coefficient then came out as 0.58 ± 0.01. Due to the greater normal force, the sheets were in more contact, increasing the coefficient of friction.

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