

**23. ročník, úloha II.2 ... break the bridge !!! chybí statistiky !!!**

What is the optimum speed of a walking person to achieve the maximum amplitude of the bridge? Define all the parameters needed and then solve.

First, let us idealize the situation by replacing the bridge with a simple string placed in vacuum, far away from any sources of the gravitational force. We can arrange for the greatest oscillations by forcing the string to vibrate with a frequency equal to the frequency of the given normal mode of the string.

Let us assume that the string vibrates at the first mode. It is clear that we should be plucking the string at its middle point with a periodic force. However, we assume that the same effect is caused by plucking the string at any point. Therefore, if we move with a velocity  $v_k$  and our step has a length  $x_k$  the induced frequency is  $v_k/x_k$ . We want this frequency to be equal to the normal mode frequency of the bridge. To obtain the formula for normal mode frequency, we can either solve the wave equation with appropriate boundary conditions or we can search the internet. The result is  $f = nc/2L$  where  $c$  is the speed of propagation of waves in the string,  $n$  stands for the given mode of vibration and  $L$  denotes the length of the string. Furthermore, the wave speed is equal to  $\sqrt{T/\lambda}$  where  $T$  denotes the tension and  $\lambda$  the linear density of the string. Parametrization of the given problem is mostly a matter of experimentation, not theory. Therefore, it can be efficiently handled by engineers.

*Jan Humplík*

`honza@fykos.mff.cuni.cz`